HEAT AND MASS TRANSFER IN DRYING OF GROUND PASTE-LIKE MATERIALS IN A COMPACT LAYER

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The paper formulates and solves the problems of heat and mass transfer in a compact layer of disperse material for the case of a drying agent supplied through the material with constant and fluctuating schedules of convective drying.

The general theory of energy and mass transfer as applied to the drying process has been correlated and developed by Luikov. However, individual problems connected with the drying of specific materials require further research.

Thus, for instance, the process of drying paste-like materials such as polymers, baking or nutrient yeast, dyes, etc., which as a rule are reduced to the form of grains or granules before drying, is complicated by the higher initial moisture content, considerable shrinkage in drying, a tendency to sticking together and aggregation at the beginning of drying, considerable nonuniformity of the fractional composition of particles, and relatively low permissible heating temperature for the material. Thus, the process requires an individual approach to each specific material.

In this connection a number of difficulties arise in calculating the drying kinetics of a given class of materials. In general practice (for instance, with conveyer driers of the PKS type), the adjustment of the process is made at the expense of the drier's output. This in turn adversely affects the general technological operation of production.

In order to regulate the process of drying, not at the expense of the output of a drying installation but by changing the parameters of the drying agent, i.e., in order to automate the process, it is necessary not only to estimate the duration of the process, but also to know the kinetics of the temperature and moisture conditions of the material.

Statement of the problem. Let the drying be performed on a continuous multistage belt drier with the material mixed at points of transfer from belt to belt. When we consider the process as being made up of individual stages (roughly, let us consider that each stage corresponds to the time the material remains on one belt), then in each stage or segment there takes place a steady-state heat exchange with a 90° cross movement of two heat carriers—the air and the material—with a negative internal source of heat in the latter.

As stated in [2], the process of steady-state heat exchange in the given case will be similar to an unsteady-state heat exchange in a stationary compact layer of disperse material, and the problem of heat exchange in the stationary layer under specific conditions may be reduced to the problem of heat exchange in a regenerative heat-exchanger apparatus with the use of appropriate differential equations.

At the beginning of an individual segment the material has a layer height h_i, the temperature of which at the moment of time $\tau = 0$ is equal to Θ_i. Air begins to flow through the layer at a temperature t_i. At the same time within the layer there appears a negative internal source of heat of intensity q. The physical constants of both heat carriers are known, and we have to find the temperature field as a function of time and the coordinates.

To solve the given problem it is necessary to maintain the following essential conditions:

1. A small temperature gradient inside the individual particles. If $\mathrm{Bi} \ll 1$, it may be disregarded in practice. (For instance, for the case of drying yeast grains with an initial diameter of 2 mm, $\mathrm{Bi} \approx 0.1$).

2. The quantity of heat transmitted from particle to particle in the direction of the air flow is negligibly small in comparison with the quantity of heat exchanged between the particles of the material and the air passing through it, i.e.,

$$-\lambda_{\mathrm{m}} \frac{\partial \Theta}{\partial y} \ll \alpha (t - \Theta_s).$$

Then we can write the following set of differential equations of partial derivatives [2]:

$$t = \Theta + \frac{C_{\mathrm{m}}}{\alpha\sigma} \frac{\partial \Theta}{\partial \tau} - \frac{q}{\alpha\sigma\gamma_{\mathrm{m}}}, \tag{1}$$

$$\Theta = t + \frac{W}{\alpha F} \frac{\partial t}{\partial \tau} + \frac{W_a h}{\alpha F} \frac{\partial t}{\partial y}. \tag{2}$$

The second term of Eq. (2), as we see from a substitution of numerical values for the worst case which can occur in practice, constitutes approximately 0.05% in relation to the third term and we can disregard it.

Let us designate

$$\varepsilon = q/\alpha\sigma\gamma_{\mathrm{m}}, \quad A = \alpha F/W_a h, \quad B = \alpha\sigma/C_{\mathrm{m}}.$$

Then a set of equations may be written in the following form:

$$t = \Theta + \frac{1}{B} \frac{\partial \Theta}{\partial \tau} - \varepsilon, \tag{3}$$

$$\Theta = t + \frac{1}{A}\frac{\partial t}{\partial y}. \qquad (4)$$

In order to solve the given set of equations, it is necessary to assign a relationship to the negative internal source of heat or, more precisely, to the complex $\varepsilon = q/\alpha\sigma\gamma_m$.

In the general case q is a function of the layer height and time. The form of this functional relationship is unknown. From the theory of drying it follows that the intensity of evaporation of moisture from a material is proportional to the partial pressure difference between the ambient vapor and the evaporation zone. On the basis of the physical meaning of the process, we can show that q is mainly dependent on the temperature of the medium and material, the dependence of which on τ and y is accounted for in the system of equations.

It is noted in [3] that during the first drying period the rate of drying depends almost entirely on the product $\alpha F(t_a - t_s)$ with $t_s = t_{wb}$. Consequently, we can assume that during the first period of drying, the value of q is proportional to the difference between the dry- and wet-bulb temperatures in air.

The second period of drying is more complicated. Luikov recommends calling it a period of rising temperature of the material and falling rate of drying [1]. In drying disperse material in a layer with a drying agent blown through the layer, the problem is further complicated by the fact that it is difficult to determine precisely where and when the first period terminates and the second period begins, since the lower strata dry faster than the upper strata.

An approximate answer to this question is obtained by periodic sampling of the material for moisture at different depths of the layer. For example, in drying yeast the experimental data show that within the stratum next to the screen the first period lasts approximately 5 min, while in the upper stratum (with periodic mixing of the material) the period (depending on the drying conditions) is several times longer. Therefore, it is of special interest to choose a single law of change of q for both drying periods.

It is known that the period of constant drying rate comes to an end at the moment when the surface moisture becomes equal to the hygroscopic moisture. This leads to the deepening of the evaporation zone within the material, while the surface temperature rises above t_{wb} and approaches the ambient temperature, i.e., t. It follows that the vapor leaves the surface of the material, and passes into the atmosphere at the surface temperature.

Thus, the drying rate decreases when the surface temperature rises, and drops to zero when the surface temperature becomes equal to the ambient temperature. In connection with this phenomenon, Krischer [4] recommends the construction of drying-rate curves based on graphical-analytical calculations of the apparent rate of drying proportional to $(t_L - t_E)$, where t_L is the ambient temperature and t_E is the temperature in the evaporation zone at the end of the period.

In the drying of thermolabile materials, to preserve the quality of the product, the process takes place at comparatively low temperatures (criterion Bi, as shown above, is small). Taking this into account, we can assume the evaporation intensity of the process as a whole to be proportional to the difference between the ambient temperature and the temperature of the material. However, this will hold fully only for the case when the material is delivered to the drier at the wet-bulb temperature. In practice, the initial temperature of the material fed Θ_i differs from t_{wb}.

Thus, for example, if $\Theta_i < t_{wb}$, then from the accepted relationship $q = q_0 (t - \Theta)$ it will follow that, at the beginning of the process, the source q at Θ_i must be greater than at t_{wb}, and this does not correspond to fact. Nevertheless, if one accepts the experimental data showing that the whole layer (of optimum height) of the material assumes the wet-bulb temperature during the first 2–3 min, it may rightly be assumed that, after an initial time lapse of 5 min, the accepted relationship must correspond to the actual nature of the process; that is, with a rise in Θ within the lower strata, when these strata pass into the second drying period, the difference $t - \Theta$ decreases and produces a diminution of the source. In the limiting case this difference becomes zero, i.e., the material reaches a moisture equilibrium and evaporation ceases. On the other hand, the source will be at its maximum at the beginning of the process (after 3–5 min) when the difference is at a maximum.

Thus, if we accept the relationship

$$\varepsilon = \varepsilon_0 (t - \Theta) \qquad (5)$$

and determine experimentally ε_0 for each individual section (of the drier belt), then, substituting these values in the expression for the temperature of the material obtained from the solution of the set of differential equations and comparing the result with the experimental data, it will be possible to judge how valid the accepted law is.

By its physical meaning the dimensionless complex $\varepsilon_0 = q_0/\alpha\sigma\gamma_m$ characterizes the interrelationship between the external and internal transfer of moisture, account being taken of the shrinkage of the material.

The coefficient of heat transfer α which enters into this complex, as shown in [5], has somewhat higher values than in the case of "pure" heat exchange due to the effect of mass exchange on heat exchange. However, if we bear in mind that when the moisture content of the material is higher, there is greater sticking together and aggregation of particles, which leads to a smaller surface of the material being exposed to heat exchange (2), and that humidification of the air as it passes through the layer of moist material brings the values of α closer to the values of "pure" heat exchange (5), we may assume that the effect of the coefficient α will be insignificant, since one phenomenon is superimposed upon the other.

With an increase in air velocity, α increases. However, at the same time evaporation is intensified, being accompanied by an increase in the specific surface σ and a decrease in the specific weight of the material γ_m. Apparently, in the future it will be possible to obtain for certain specific materials an empirical relationship for determining ε_0, depending on the schedule parameters and the initial moisture of the material for the individual stages.

The coefficients A and B, which account for the effect of external heat exchange and material shrinkage on the kinetics of heating, are generally functions of the height of the layer and the time. The solution of a given problem with coefficients having two-parameter variables will have a very complex form and would be difficult to apply. In any case, we would have to express these coefficients through τ and y by using the empirical relationships obtained by experiment.

Special tests to investigate the shrinkage of materials of the class under consideration showed that (by breaking up the process into 5–6 stages) it is possible to assume with sufficient precision for the individual stages that the coefficients A and B are constant and equal to the mean effective values.

Solution of the problem. 1. The case of air entering at a constant temperature. By substituting (3) in (4), account being taken of (5), we obtain

$$\frac{\partial^2\Theta}{\partial y\partial\tau}+A\frac{\partial\Theta}{\partial\tau}+B(1+\varepsilon_0)\frac{\partial\Theta}{\partial y}=0. \qquad (6)$$

Let us designate

$$B(1+\varepsilon_0)=b.$$

The hyperbolic equation of the second order thus obtained is solved by the Laplace-Carson method of integral transformations for the following boundary conditions:

$$\Theta|_{\tau=0}=\Theta_i, \quad t|_{y=0}=t_i. \qquad (6a)$$

The solution has the form

$$\Theta=\Theta_i+(t_i-\Theta_i)b\exp(-Ay)\times$$
$$\times\int_0^\tau\exp(-b\tau)I_0(2\sqrt{Aby\tau})\,d\tau. \qquad (7)$$

By substituting (7) in (3), we obtain the formula for determining the air temperature:

$$t=\Theta_i+(t_i-\Theta_i)\exp(-Ay)[\exp(-b\tau)I_0(2\sqrt{Aby\tau})+$$
$$+b\int_0^\tau\exp(-b\tau)I_0(2\sqrt{Aby\tau})\,d\tau. \qquad (8)$$

The integral present in formulas (7) and (8) may be calculated either graphically or analytically by a series expansion of the Bessel function. After integration we obtain here a rapidly diminishing series. Also, by limiting numbers to the first three terms, the error will not amount to more than 1%.

For a more rough evaluation (increasing the error to 10%), we may limit ourselves to the first term alone, i.e., we assume $I_0(2\sqrt{Aby\tau})=1$. Then

$$\Theta\approx\Theta_i+(t_i-\Theta_i)\exp(-Ay)[1-\exp(-b\tau)], \quad (9)$$

$$t\approx\Theta_i+(t_i-\Theta_i)\exp(-Ay). \qquad (10)$$

2. The case involving fluctuation. At the present time ever-increasing use is being made of a fluctuating schedule in drying thermolabile materials, as this method makes it possible to obtain a dried product of higher quality. The problem considered above may also be solved for the case of drying on a fluctuating schedule.

Let the air be delivered under the drier screen (belt) at a fluctuating temperature, varying from t_{max} (heating) to t_{min} (cooling) with a frequency $\omega = 2\pi/T$, where T is the period of fluctuation. Then, if we designate the amplitude by $k = (t_{max} - t_{min})/2$ and assume that, because of the inertia of the system, the temperature changes do not proceed in jumps but follow the sinusoidal law, the boundary condition (6a) for Eq. (6) will take the following form:

$$t|_{y=0}=t_{av}+k\sin\omega\tau,$$

where

$$t_{av}=(t_{max}+t_{min})/2,$$

and the solution will be

$$\Theta=\Theta_i+b\exp(-Ay)\cdot$$
$$\cdot\Big[(t_{av}-\Theta_i)\int_0^\tau\exp(-b\tau)I_0(2\sqrt{Aby\tau})\,d\tau+ \qquad (11)$$
$$+k\int_0^\tau\sin\omega(\tau-\xi)\exp(-b\xi)I_0(2\sqrt{Aby\xi})\,d\xi\Big],$$

$$t=\Theta_i+(t_{av}-\Theta_i)\exp(-Ay)\Big\{\Big[\exp(-b\tau)I_0(2\sqrt{Aby\tau})+$$
$$+b\int_0^\tau\exp(-b\tau)I_0(2\sqrt{Aby\tau})\,d\tau\Big]+k\Big[\int_0^\tau\sin\omega(\tau-\xi)\times$$
$$\times\exp(-b\xi)I_0(2\sqrt{Aby\xi})\,d\xi- \qquad (12)$$
$$-\omega\int_0^\tau\cos\omega(\tau-\xi)\exp(-b\xi)I_0(2\sqrt{Aby\xi})\,d\xi\Big]\Big\}.$$

If we limit ourselves to the first term of the series expansion of the Bessel function, as we did in the first case, then, after completing the integration, we obtain

$$\Theta\approx\Theta_i+\exp(-Ay)\Big\{(t_{av}-\Theta_i)[1-\exp(-b\tau)]+ \qquad (13)$$
$$+\frac{kb}{b^2+\omega^2}[b\sin\omega\tau+\omega\cos\omega\tau-\omega\exp(-b\tau)]\Big\},$$

$$t\approx\Theta_i+\exp(-Ay)\Big\{(t_{av}-\Theta_i)+$$
$$+\frac{k}{b^2+\omega^2}[(b^2-\omega^2)\sin\omega\tau+2b\omega\cos\omega\tau]\Big\}. \qquad (14)$$

From a comparison of formulas (9), (10), (13), and (14), it follows that the current temperatures of the material and air within the layer, in the case of drying on constant and fluctuating schedules (with $t_i = t_{av}$), differ by the presence of an additional term, in the case of the fluctuating schedule. This term contains $\sin \omega\tau$ and $\cos \omega\tau$ combined with an exponent which takes account of the effect of the height of the layer. This points to the fact that a change in temperature with an increase in the height of the layer will lag in phase as compared with the change in air temperature at the inlet. This picture of temperature redistribution with respect to the height of the layer, which is experimentally confirmed, leads to more uniform drying of the material.

An analysis of formula (11) shows that the average temperature of the material in the case of fluctuations should be somewhat lower than in the case of drying without fluctuations, since in formula (13), as compared with formula (9), there is an additional negative term which does not contain trigonometric functions (b is always greater than 0).

A comparison of the juxtaposed cases is also of interest with respect to the intensity of evaporation, i.e., with respect to the magnitude of the internal source.

Without fluctuation

$$\varepsilon = \varepsilon_0 (t_i - \Theta_i) \exp(-Ay - b\tau). \qquad (15)$$

With fluctuation

$$\varepsilon = \varepsilon_0 \left\{ (t_{av} - \Theta_i) \exp(-Ay - b\tau) + \frac{k\omega}{b^2 + \omega^2} \exp(-Ay) \times \right.$$
$$\left. \times \left[(2b - 1) \cos \omega\tau - \omega \sin \omega\tau + \exp(-b\tau) \right] \right\}. \qquad (16)$$

In formula (16) there is an additional term $\exp(-b\tau)$ which always has a positive value. Consequently, in the second case the process of evaporation will be more rapid, since the trigonometric functions reflect the effect of fluctuations of the source relative to a certain average value and do not produce any essential effect on the intensification of the process in the segment as a whole.

Here we must emphasize the fact that the basic importance of fluctuation lies not in the intensification of the process, since the increase for the process as a whole amounts only to approximately 5–10%, but in obtaining a product of higher quality. Each specific product will have its own optimum fluctuation schedule because, strictly speaking, the empirical coefficient ε_0 will also depend on the period and amplitude of the fluctuations.

Then it is possible, after the integration of formulas (15) and (16) with respect to τ and y, to obtain a relationship for determining the quantity of evaporated moisture, which in turn will permit us to determine the drying time for the selected schedule.

NOTATION

α) heat transfer coefficient; t, Θ) variable temperatures of air and material in layer; h) total height of layer; y) variable thickness of layer; τ) drying time; W) water equivalent of air filling the spaces between particles of material; W_a) water equivalent of air passing through the layer; F) surface area of material; σ) specific surface of material; q) negative internal heat source. Subscripts: a) air; m) material; i) initial value of parameter; s) surface; av) average value of parameter.

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